MEASURES OF LONG DISTANCES

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Abstract

The theory of Earth moving around the Sun had, in Galileo age, serious problems. One of these was that, if the Earth is moving, you could observe stellar parallax: you could observe different positions of "nearer" celestial bodies against the far away stars. And this was not verified. No parallax? Then, no heliocentric theory! This sentence is similar to the famous advertising: "No Martini? No party!" Scientific proposals must be supported with experimental observations (soon or late). Galileo explained why parallax was not visible: because of big distances among celestial bodies and angular resolution. The Universe was bigger than assumed by Galileo contemporary people and celestial bodies too far from the Earth for technological tools of 1600.

We wish to understand, in this very practical workshop, the problem of distance, angular size and error with angular distances. Parallax has been a tool to investigate the Universe for ancient astronomers but it is limited by the resolution of angles.

In the first part of the workshop we wish to practice with angular sizes measurements with a simple device hand made. In the second activity we measure the height of a tower, a building (at known distances)… using one eye and a stick. In the third one we measure the diameter of the Sun using tubes and aluminium foils with a pin hole.

Perhaps at the end of the activity young students appreciate better math, trigonometry or simple geometric assumptions because they amuse themselves with sticks and viewers!

INTRODUCTION

The revolutionary idea of Copernicus was, to be honest, a hypothesis to simplify the model of solar system, not a proposal in accordance with a physical interpretation of phenomena observed. Before Kepler and Newton studies it will be impossible to understand and "prove" the heliocentric model.

The Universe vision of Copernicus is still a medieval vision with aristothelic-like suppositions.

The revolution of Galileo was first the method. The interpretation of phenomena observed at least with any human contamination, separating different components of a phenomenon to better understand; to use numbers, geometric lines and figures to interpret the results. Nature is a book written with numbers and figures… In any case the hypothesis have to be confirmed by data.

So the problem of lack of parallax was a main point of discussion in scientific circles in Galileo years. Galileo tried to observe by himself parallax during six months observations of the nocturnal sky but he could not succeed.

We wish to handle with angular distances and angular sizes of objects far from us, to understand why it was not possible in 1600 to verify good hypothesis.

Nowadays astronomers use photos from the space to verify parallax with high precision.

FIRST ACTIVITY

To make a viewer to measure physical-real size of distant object when we know the distance to the object; or, on the contrary, to calculate the distance knowing the sizes.

In fact a distant of an unknown object could appear really difficult to interpret if we do not know the real size and we know that an object can seem to be much bigger or smaller than it is if we compare it in a wrong way with similar distant different objects.

In the following activity we will use a simple formula to obtain the real-physical size of the object.

Before beginning the construction of the device, the angular size viewer, we remember:

$$360^\circ = 2\pi \ rad = one\ full\ circle$$

$$1\ radian = \frac{360^\circ}{2\pi \ rad} \approx 57^\circ$$

We make the “Angular size viewer” following the little picture in the model (Figure 2) and the steps: cut around, all along the outline of the model; fold up along the dot line, the squared one with the hole and the strip with 10, 20… degrees marked on; glue as written and make the hole with the point of a pencil.

Now let’s calculate long distance and/or object’s size using angular size measurements.

After, taking two or three objects of a known size L (e.g. a 30 cm ruler or a 70 cm length table…) we put us at two or three different distances marking the different positions on the floor.

Using the viewer we measure the angular size of the object observing from the hole by accurately coinciding the 0° with the beginning of the ruler, the table, etc. (Figure 2). For example if we obtain 6° for the 30 cm ruler at a certain distance we call it α**.**

Now we use the formula:

$$D = 57{,}3^\circ \ \frac{L}{\alpha}$$

In our case, distance is equal to 57,3 × 30 cm divided by 6.

We can verify that the distance we obtain (2,8 m) is the same we marked on the floor. So it happens for the other marked positions.

This method works only for angles up to 20° and for distances no more than 4 or 5 meters (for little objects). For more than 5 meters, the angular size of a 30 cm size object is difficult to measure with a satisfying accuracy.

But if we know the size of a big object, for example a car or a building, we can use the procedure to calculate the unknown distances. The same procedure can be used to calculate the distance to the Moon and verify it with a school-book data. Of course we can calculate physical size knowing the distance. So we practise with size (physical size if you prefer) angular size and distances.

To understand the relation between *D* and *L* indicated above, we must consider the approximation for small angles. This part can be proposed to students that are learning trigonometry.

Draw an angle α whose base is *L* and a distance *D* subtending the angle (Fig. 1). The perpendicular from the apex to the base permits us to say that:

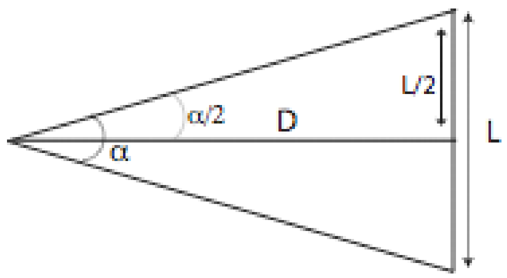


Figure 1

$$\tan\frac{\alpha}{2} = \frac{L/2}{D} = \frac{L}{2D}$$

$$L = 2D\tan\frac{\alpha}{2}$$

If we assume that when the angle is very small $\tan\alpha \approx \alpha$ we have:

$$L = 2D\frac{\alpha}{2} \quad \rightarrow \quad L = D\alpha$$

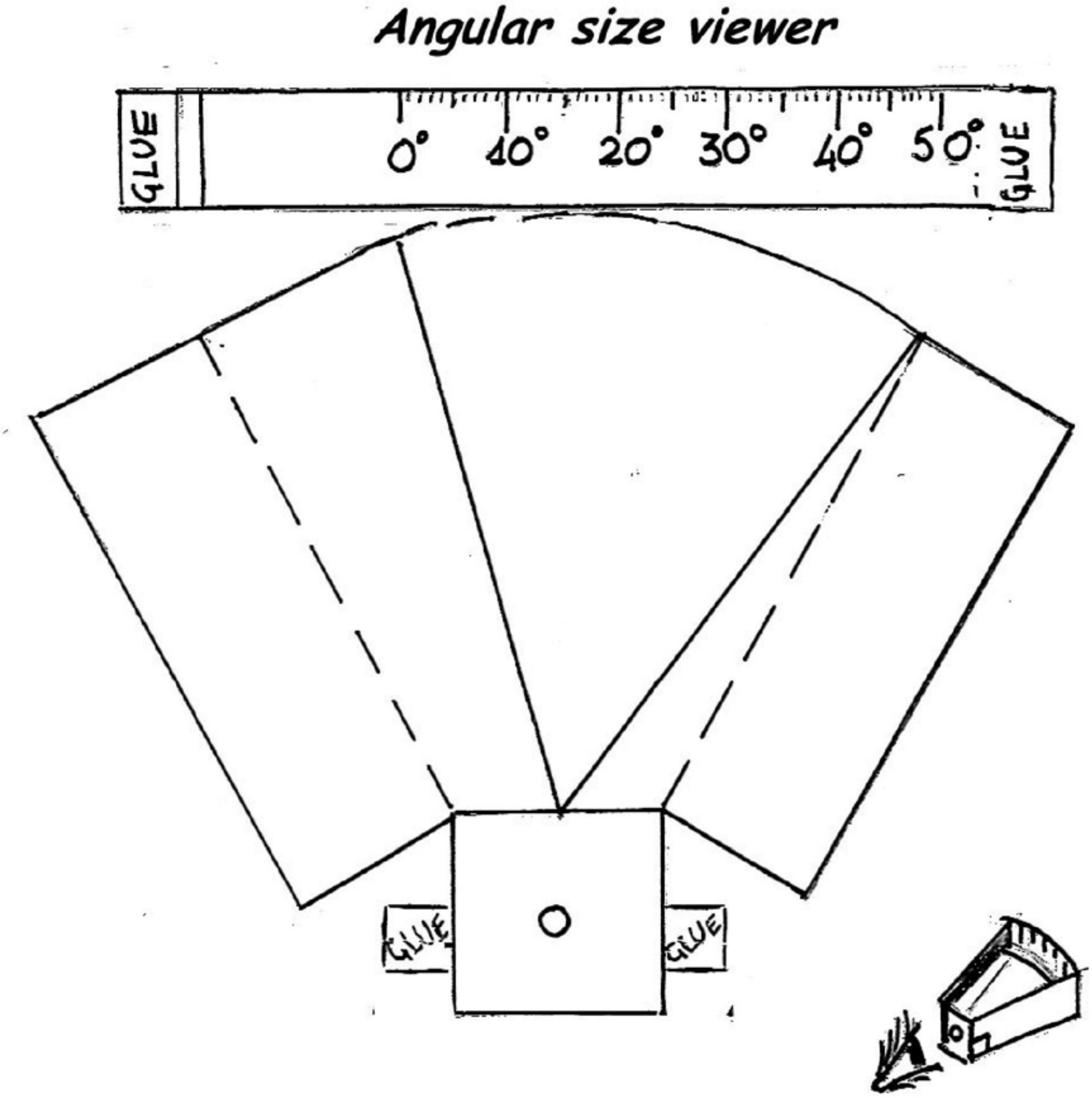
To be sure that the approximation for small angles is correct we can compare the two calculations: Calculation of the angle subtended by the Sun a) without and b) with approximation.

<u>*Data*:</u> Diameter of Sun: 698 000 Km; Distance to the Sun: 150×10^6 km.

a) $\alpha = 2\ \arctan\frac{2\times 6{,}98\times 10^5}{2\times 1{,}5\times 10^8} = 9{,}3066\times 10^{-3}\ rad$ (without approximation)

b) $\alpha = 2\ \frac{6{,}98\times 10^5}{1{,}5\times 10^8} = 9{,}30667\times 10^{-3}\ rad$ (with approximation)

Now we understand the formula for our device: $D = \frac{57{,}3\ L}{\alpha}$, the factor 57,3 converts degrees in radians.



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OBJECT	**SIZE** L	**ANGULAR SIZE** α	**DISTANCE** d

Figure 2. The angular size viewer

SECOND ACTIVITY

<u>*Activity*</u>: To measure the height of a tower or building.

<u>*Material needed*</u>: two or three sticks (80 cm, 100 cm…) and two volunteers.

We look at a building, a tree, a tower… We move from it and we put a stick vertically in the floor, or in the grass by the first volunteer at a certain distance; the second volunteer moves on from of the stick to a point where laying on the floor he/she can see with one eye the coincidence of the top of the building, tree …, with the top of the stick. See the Figure 3.

Now we mark these positions and measure the distance: stick-eye (*d*) and building-eye (*D*). Looking at Figure 3 we can write the proportion:

$$\frac{L}{l} = \frac{D}{d}$$

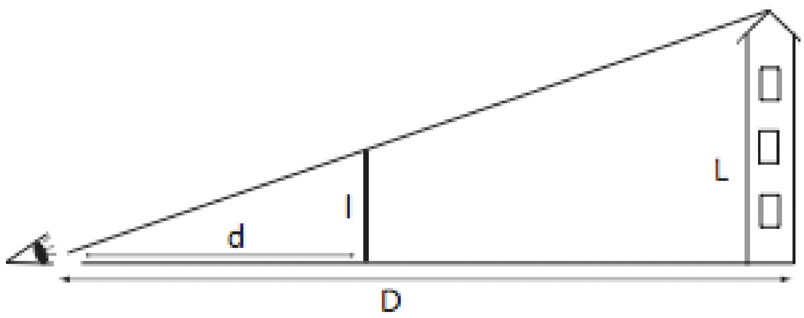

Figure 3. Proportion building- stick- distances

Where L is the height of the building and l is the length of the stick.

Using different sticks we can calculate the medium value of the different results for L.

THIRD ACTIVITY

To measure the dimension of a long distance object it is not necessary to use parallax or to measure an angle that is, of course, difficult because of big errors in little measures of angles. We can simply use the similitude of triangles and measure the length of the dimension of a figure. The error in this case is minor. The proportion is the same of the previous activity. What we wish to underline is that nowadays we have measurements with great precision and we use it to find other measurements. On the contrary in ancient time scientists did not have real measurements of long distances object: they supplied to this using really very brilliant methods with which, in a sort of chain, they used the result obtained in an observation modality to achieve results in another one. So again we confirm that methodology is the best to increase knowledge better than technology. The same is in education!

<u>*Activity*</u>: To measure the diameter of the Sun.

<u>*Material*</u>: Cardboard or plastic tubes of different (length and diameter) sizes; aluminium foil, a pin to make a hole, a semi - lucid paper sheet, adhesive tape or glue.

Instructions: For each tube you have to fix (using glue or adhesive tape) an aluminium foil at one end and a semi lucid paper sheet on the other end. Make a pin hole in the centre of the aluminium foil. This part of the tube has to be turned to the Sun in a way so that the sunbeams enter the tube. To do this, you move the tube gently looking on the floor (with your back to the Sun) the shadow shape of the tube: when you can see only the bottom of the tube (using a white paper sheet a few centimetre from the tube) the alignment is right. If you look well, you see a spot that is the image of the Sun on the semi-lucid paper. Now you have to fix the position and this is the most difficult: you can use a tripod or any thing fits for. Mark the spot with a pen with most accuracy and compare the circle with a millimetre –paper in order to measure the diameter of the spot. Students will notice that the diameter of the tubes has no influence; on the contrary more length means a bigger spot. With a tube of one-meter length, the spot is about 9 millimetres.

Figure 4. The device for the observation of the Sun and its usage

We calculate the diameter of the Sun using proportion $\frac{D}{TS} = \frac{d}{l}$ that we obtain from the Figure 5.

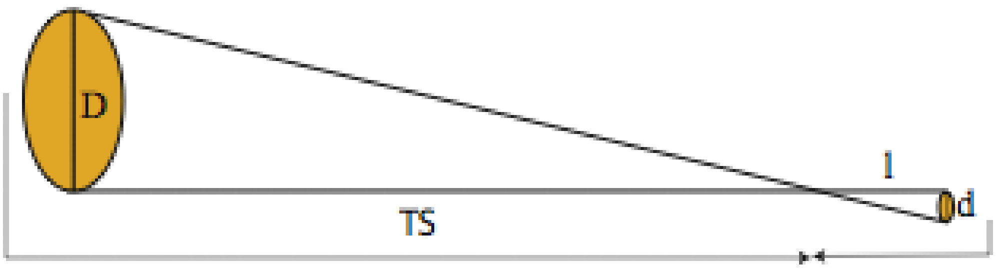

Figure 5. Proportions between diameters and distances

TS: distance to the Sun *d*: diameter of the mark of Sun

D: diameter of the Sun *l*: length of the tube

The measurement obtained has error minus than 10% using millimetre paper for the *d* measure and using the distance of the day for the *TS* measure.

SAFE WARNING!

To find the image of the Sun you put your back to the Sun and look to the image in a white piece of paper near the end of the tube. The students in the photos are doing the activity in a cloudy day (in Torino it often happens!): it is a good safe-idea if the Sun is weakly visible. No danger for eyes and good results in any case.

Figure 6. Students using the tube for calculating the diameter of Sun

In the other photo (Figure 4, right) you see how to use the device in order to facilitate the research of the image of the Sun: putting the tube on a window and moving the window it is easier to block the image! Also in this case the heavy curtain around the device guaranties the safety.

HOW BIG WAS THE UNIVERSE IN ANCIENT PERCEPTION?

How could ancient philosophers-scientists have an idea of the dimensions and distances of celestial bodies like Moon, Sun and planets? With skill and with an appropriate methodological approach.

The dimension of Universe in the perception of ancient Greek people (300 years B. C.!!) are indicated in the following scheme (Table 1):

Table 1

	In unità r_T (unit r earth)	In stadi	Differenza rispetto alle misure attuali error
Raggio terrestre	1	40.000	10%
Dist. Terra-Luna	70	$3 \cdot 10^6$	10%
Dist. Terra-Sole	1350	$5{,}5 \cdot 10^7$	95%
Raggio Luna	1/3		10%
Raggio Sole	6,5		95%

If we are surprised to see the accuracy of some of these measures we must remember that in Pythagoras philosophy the number and the "*Metrica*" ("Measure of things") were the core of the existential problem. In any case we are surprised and we wish to understand how they got the results.

It is also good to remember to our students that Aristarchus proposed the heliocentric theory in III century B.C. It was an hypothesis. Aristarchus also investigated about the dimension of solar system.

He investigated about distances Earth-Moon and Earth-Sun in the sense that he tried to evaluate how many times these distances are respect the Earth radius. He obtained the ratios between the distance Earth-Sun and Earth-Moon.

He obtained it with a very elegant method using two procedures. First he used the position of the three bodies (Earth, Moon, Sun) in the time when the Moon is at its first quarter phase. We have a rectangular triangle and the method is known as *dichotomy method*. See the Figure 7.

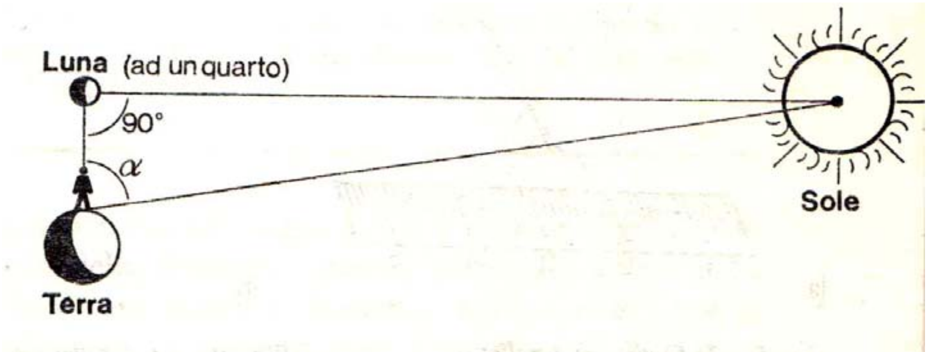

Figure 7. The Moon at its first quarter phase

Unfortunately the second angle *alfa* was measured by Aristarchus as 87° but it is 89° 50'. The difference lead to an error of course but the method is very elegant.

The distance Earth-Sun is not 20 times but 390 times the distance Earth-Moon!

The second procedure used by Aristarchus was to measure the time that elapses when the Moon passes through the shadow cone of the Earth during a Moon eclipse. With some trigonometric calculations he obtained that the distance Earth-Moon is 70 times the Earth radius. See the Figure 8.

When Eratosthenes obtained the first measure of Earth radius with the famous experiment in which he compared the angle of sunbeams in Siene and in Alexandria in the day of solstice (Figure 9), it was possible to obtain from the Aristarchus's ratios the dimension of the universe synthesized in the scheme of the Table 1.

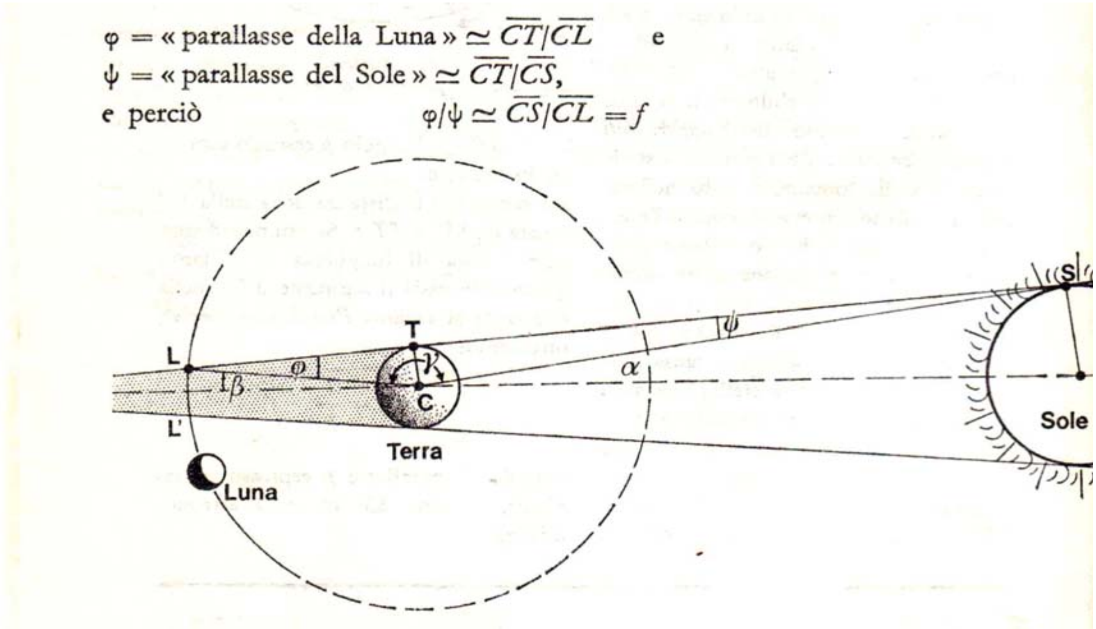

Figure 8. The Moon crossing the shadow cone of the Earth during a Moon eclipse

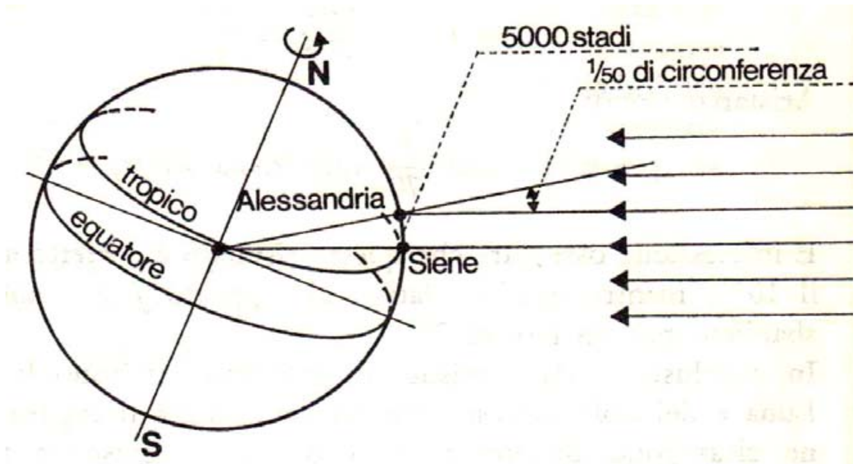

Figure 9. Determination of the Earth’s radius by Eratosthenes

It is possible to repeat Aristarchus procedures and also Eratosthenes measurement of the Earth radius but in this workshop we propose simple activities to let the student understand that with very poor material it is possible to achieve good results. The goal is to understand that with a good accuracy of known measurements of a distance (for example Sun-Earth) we can easily calculate the diameter of the Sun that can be seen like a challenge if we propose it to our students at the beginning of a physics lesson!

We must underline with students that nowadays we have a lot of data, in Aristarchus time not and in Galileo time? Was the situation of experimental data better in 1600 than 300 years before Christ birth? Of course yes if we think of the huge amount of data collected by astronomers one for all, for example, by Ticho Brahe, but not so much because of the philosophical approach of the cosmological problem. But this is another story!

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