LATITUDE GNOMON AND QUADRANT FOR THE WHOLE YEAR

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Abstract

In this workshop, we examine the correlation between our latitude and the altitude of the Sun at noon at different dates. Using this correlation, we build a quadrant with an adjustable scale for finding our latitude by observing the noon altitude of the Sun. A Viking gnomon for the same purpose is demonstrated. The students work with the concepts of declination, altitude, seasons and latitude.

THE VIKING GNOMON

To find their latitude when sailing, Vikings used a gnomon, whose height they adjusted by the date, so that the length of its shadow showed the latitude (Fig. 1). The gnomon was always vertical and its circular base horizontal despite the position of the ship, because the device floated in a bowl of water (Fig. 1, insert).

If you measure the altitude of the Sun at noon at the date of the vernal or autumnal equinox and subtract the result from 90^o, you get your latitude.

At other dates, you have to subtract Sun's declination from the measured altitude – or adjust the length of the gnomon to get the same shadow length as at equinoxes. In the latter case, you can read your latitude from the length of the shadow. You find δ_t, the declination of the Sun during the year in Table 1.

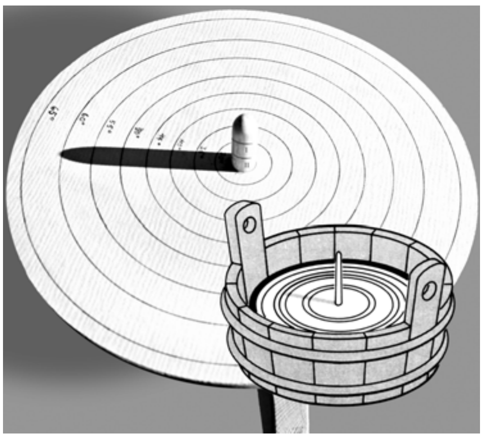

Figure 1. A model of the Viking gnomon. The gnomon is adjusted by date; here 15.2. The tip of the shadow at noon shows the latitude, here 60^o

Date	δ_t	Date	δ_t	Date	δ_t	Date	δ_t	Date	δ_t	Date	δ_t
1.1.	-23^o	2.3.	-7^o	1.5.	$+15^o$	30.6.	$+23^o$	31.8.	$+9^o$	31.10.	-14^o
11.1.	-22^o	12.3.	-4^o	11.5.	$+18^o$	10.7.	$+22^o$	10.9.	$+5^o$	10.11.	-17^o
21.1.	-20^o	21.3.	0^o	21.5.	$+20^o$	20.7.	$+21^o$	23.9.	0^o	20.11.	-20^o
31.1.	-18^o	1.4.	$+4^o$	31.5.	$+22^o$	31.7.	$+18^o$	30.9.	-3^o	30.11.	-22^o
10.2.	-15^o	11.4.	$+8^o$	10.6.	$+23^o$	10.8.	$+16^o$	10.10.	-7^o	10.12.	-23^o
20.2.	-11^o	21.4.	$+12^o$	21.6.	$+23.4^o$	20.8.	$+12^o$	20.10.	-10^o	21.12.	-23.4^o

Table 1. Approximate declination of the Sun. during a year

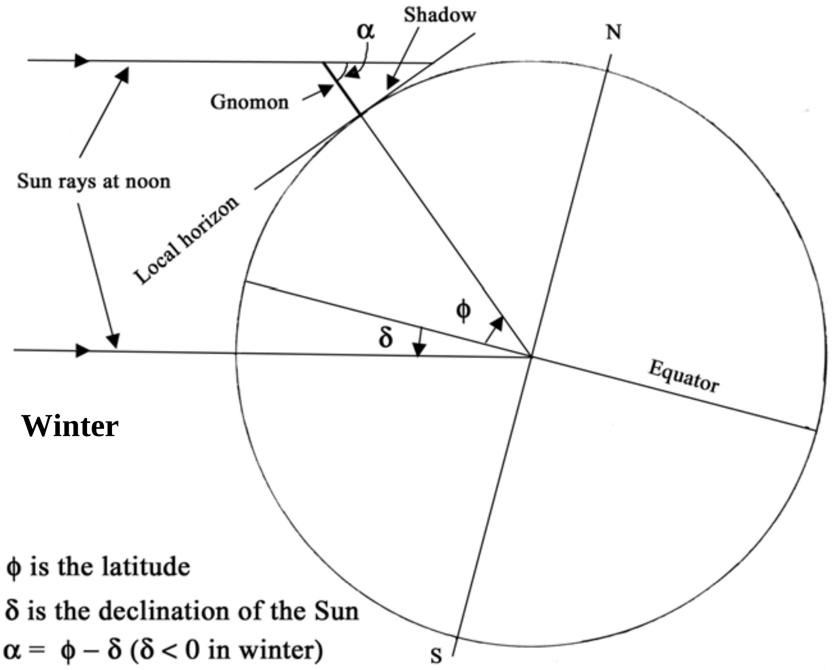

Figure. 2 The geometry of using a gnomon to find your latitude. In summer half of the years, the north pole is inclined towards the Sun ($\delta > 0$). In winter, as here, the north pole is inclined away from the Sun ($\delta < 0$) as in this figure; at equinoxes, the axis of the Earth is perpendicular to the Earth-Sun line ($\delta = 0$). For practical reasons we use the angle α between the sunray and gnomon instead of Sun's altitude. $\alpha = \phi - \delta$, and especially at equinoxes $\alpha = \phi$

To construct a Viking gnomon, first decide the height h_e of your gnomon at equinoxes (Fig. 3). Now you can find the radiuses r_ϕ for different latitudes:

$$r_\phi = h_e \tan\phi$$

Calculate r_ϕ for every 5^o from latitude 30^o to 70^o and draw the circles.

Then use one value of r_ϕ, for example $r_{45} = h_e$, to find the gnomon height h_t for other dates:

$$h_t = \frac{r_{45}}{\tan(45^o - \delta_t)} = \frac{h_e}{\tan(45^o - \delta_t)}$$

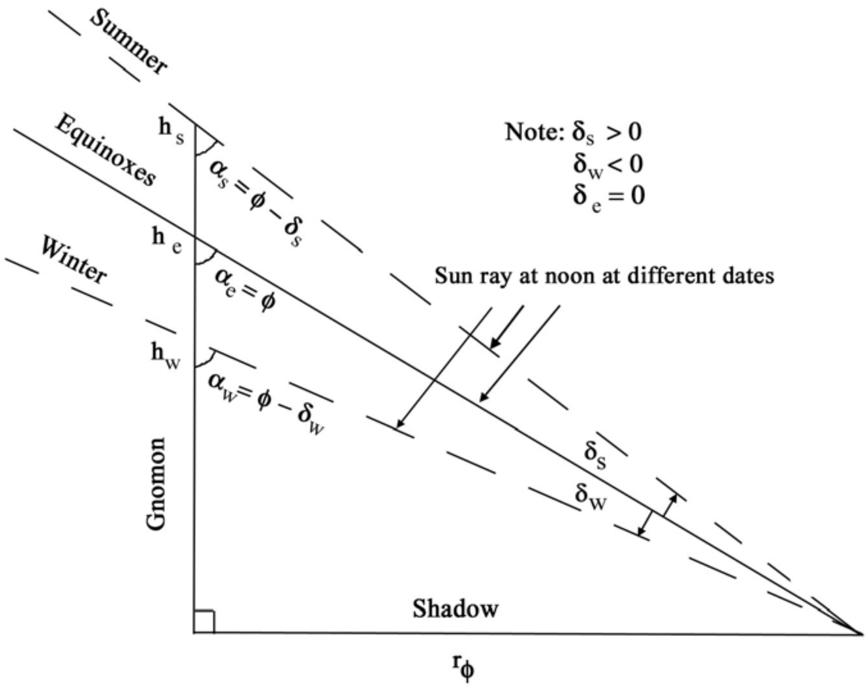

Figure 3. Calculating the gnomon height and shadow length

Use the Table 1 to find δ_t, do the calculation for the dates you think necessary – every month is enough in a crude instrument like this - and mark the gnomon.

You can save the trouble of calculations and copy Figure 4 to make the Viking gnomon of a stick and a piece of cardboard. Look at Figure 1. Note: mark the gnomon heights from the winter solstice to summer solstice and from the summer solstice to winter solstice on different sides of the gnomon.

You will find, that using the Viking gnomon is practical for latitudes 35^o to 70^o and for dates between vernal and autumnal equinox – and these are just the latitudes and dates the Vikings mostly sailed. The instrument is not very accurate. If the Vikings were going to sail, say, from southern Norway to the southern tip of Greenland, a little less than 60^{o}N, they sailed first to about latitude 61^{o}N, then due west until they saw the eastern coast of Greenland. They know, that they were north of their destination, and they had to sail south along the coast to find their destination at about 60^o N. Well, sometimes it was stormy and cloudy, and they missed their goal. This way they accidentally found America 500 years before Columbus.

AN ADJUSTABLE LATITUDE QUADRANT

The idea of the Viking gnomon can be adapted to a latitude quadrant with a movable latitude scale that can be adjusted by date, Figures 5 and 6. First, mark the latitude scale and date scale so, that the instrument shows the latitude, when its sight is pointed to the noon Sun at equinoxes. Then, add or subtract the declinations for other dates in the date scale, considering that in the winter part of the year the midday Sun is lower, in the summer part higher than at equinoxes.

Build your own latitude quadrant using a copy of Figure 6. Glue the parts on cardboard, cut them and connect the scales with a 2 mm bolt and nut. Pierce a 1 - 2 mm hole in front part of the sight, bend and glue the sight in the date scale (see Fig. 5. Use a length of thread and a M6 nut to make the plumb line.

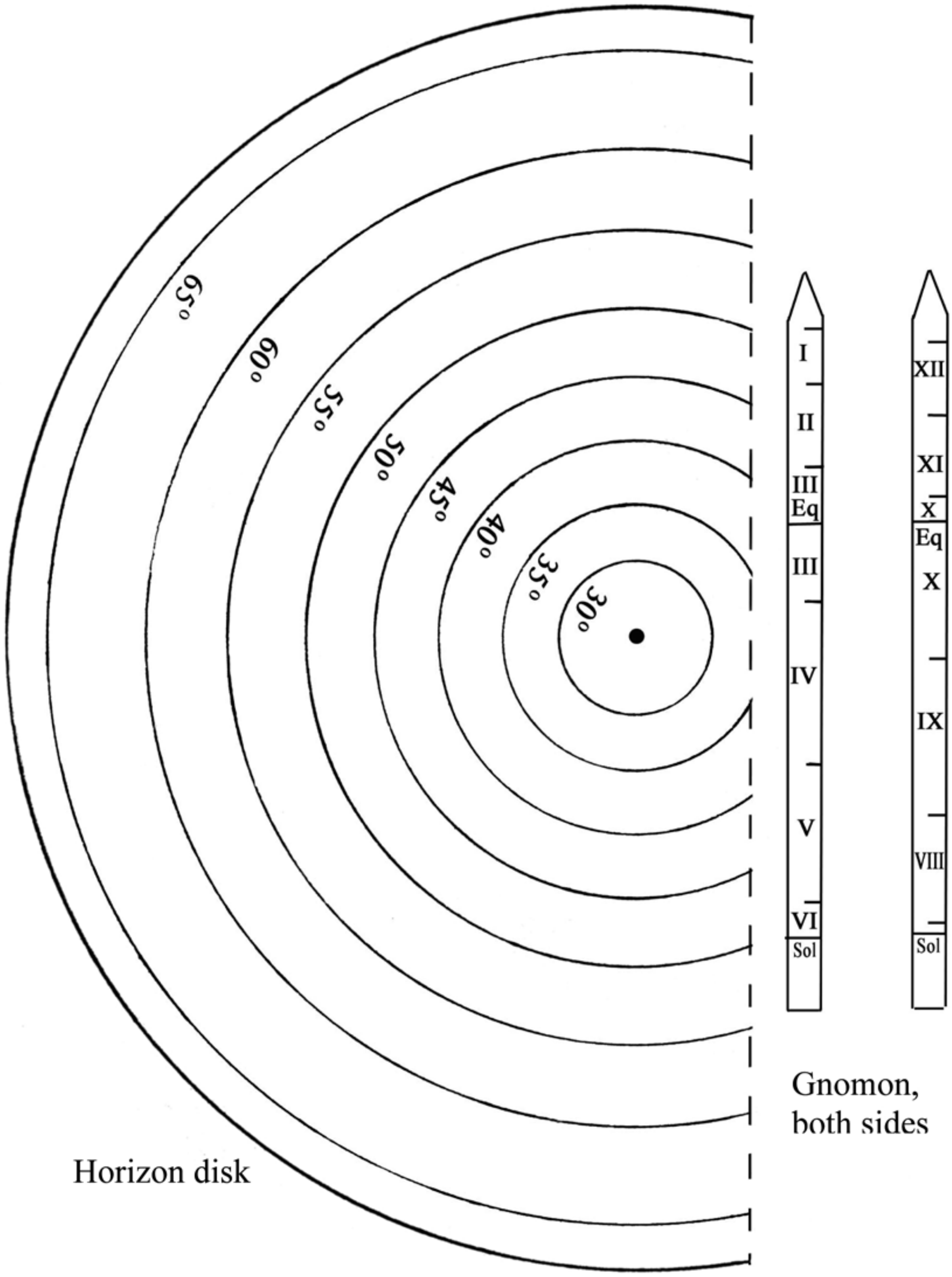

Figure 4. The parts of a Viking gnomon. Paste the horizon disk on cardboard. It is not necessary to have a circular horizon disk, but you can complete the circles, if you want to. Make the gnomon of a suitable stick and make the markings on opposite sides of it.

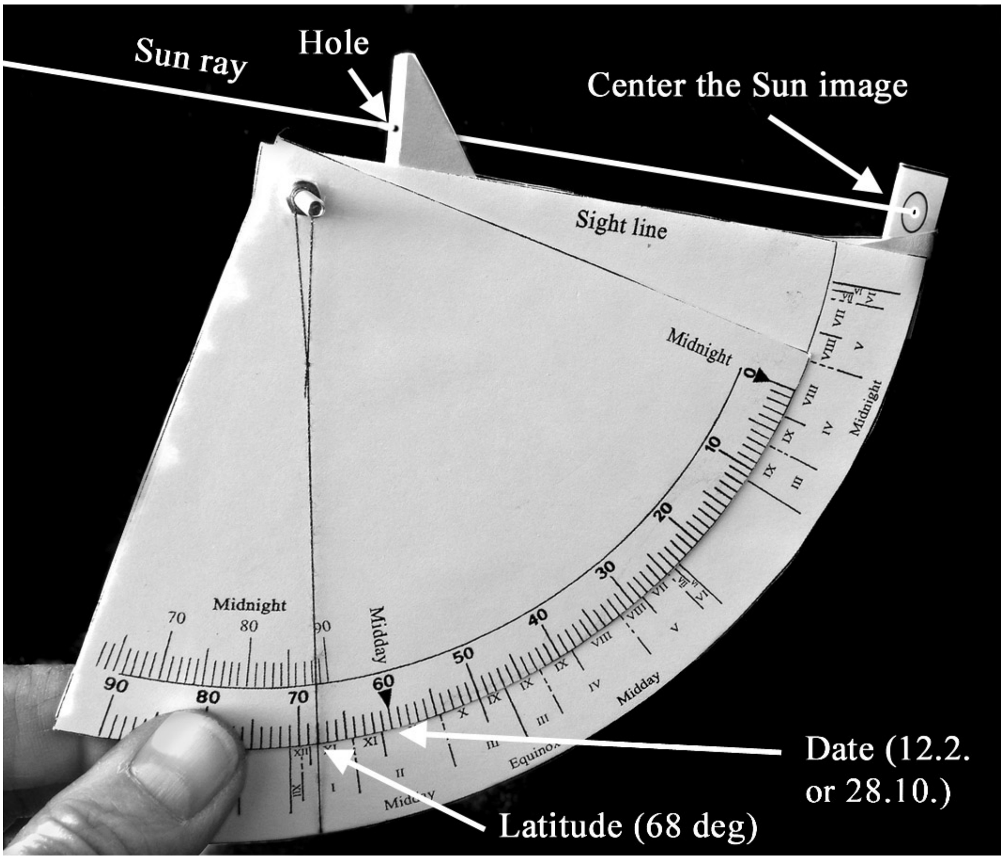

Figure 5. Latitude quadrant in use in Enontekiö, where the 6th EAAE Summer School 2002 took place

In use, first align the mark “Midday” in the latitude scale with the date, then point the sight to the Sun at noon and read the latitude from the latitude scale. Remember, that local noon in most places is not at 12:00 o’clock. If you do not know the exact time, find it observing the readings on the latitude scale around 12 o’clock. The smallest latitude reading is your latitude. The other possibility is to find the moment when the Sun is in the meridian, i.e. south.

The quadrant has a little speciality: you can use it at midnight, if you are north of Arctic Circle around midsummer. Align the triangular “Midnight” marking in the latitude scale with the date in the midnight scale and read the latitude from the midnight latitude scale. Because the Sun is very low, you have to correct for refraction. You find the refraction from the curve printed in the latitude scale. Subtract the given angle from your reading to get your correct latitude.

You can use the instrument to demonstrate the altitude of the Sun at noon at different latitudes. Just set the date and turn the instrument so, that it shows the selected latitude. The sight shows the altitude of the Sun. You can make this demonstration at a suitable time of the day, using the Sun that is lower than at noon. This works only to demonstrate the situation in latitudes north from your location, though.

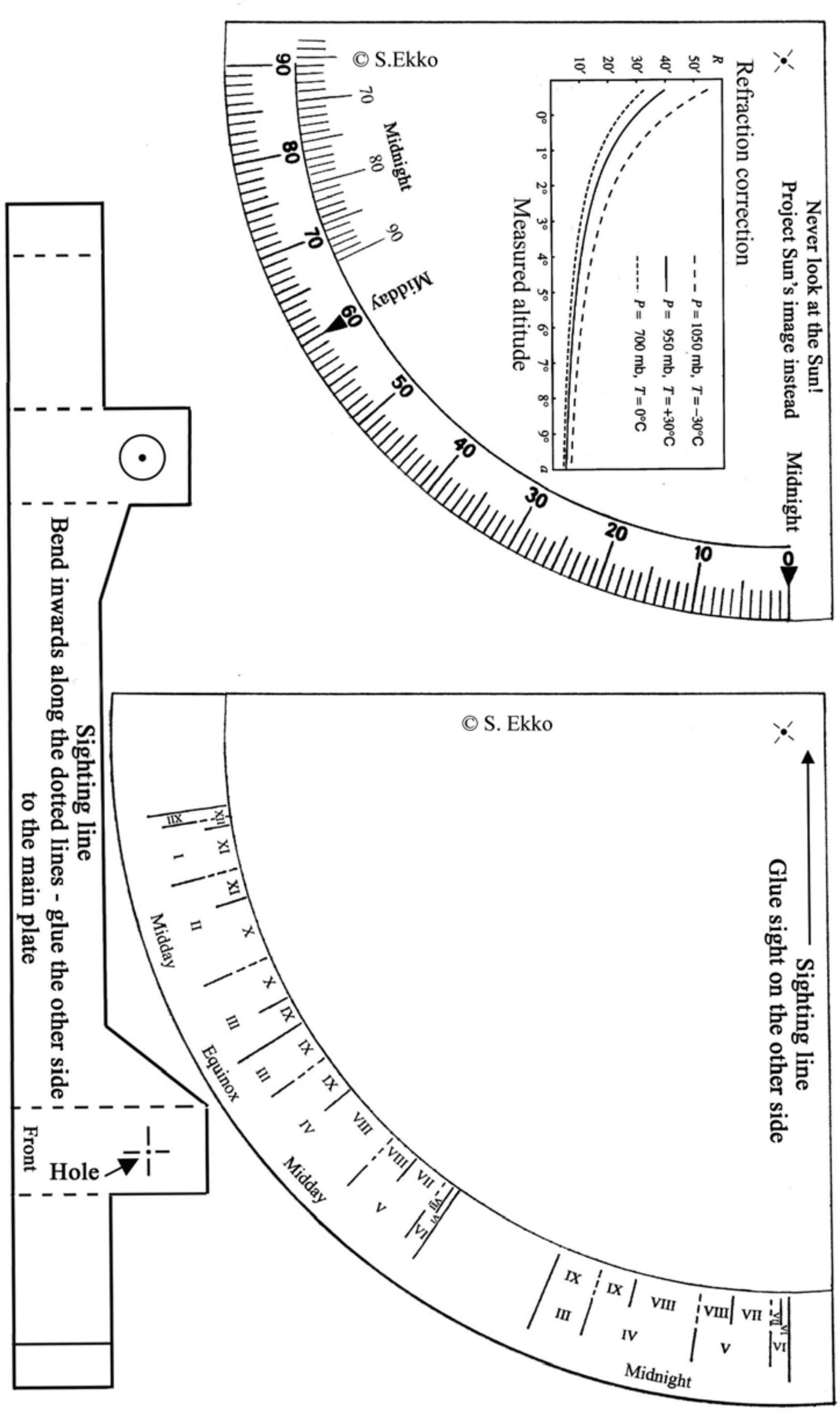

Figure 6. Parts of the latitude quadrant. Do not look at the Sun! Centre the image of the Sun formed by the front hole of the sight on the circle in the rear part (Fig. 5)

EXAMPLES

1. The Viking gnomon

Let the gnomon height at equinox be h_e = 10 cm.

In latitude 60^o, the shadow length at equinox is:

$$r_\phi = h_e \tan\alpha = h_e \tan\phi = 10cm \times \tan 60^o = 17.3cm$$

and in latitude 40^o :

$$r_\phi = h_e \tan\alpha = h_e \tan\phi = 10cm \times \tan 40^o = 8.4cm$$

and so on for other latitudes.

The gnomon height at the summer solstice is:

$$h_t = \frac{r_{45}}{\tan(45^o - 23.4^o)} = \frac{10cm}{\tan 21.6^o} = 25.3cm$$

and at 30.9.:

$$h_t = \frac{r_{45}}{\tan(45^o - (-3^o))} = \frac{10cm}{\tan 48^o} = 9.0cm$$

Calculate enough h_t-values to get the accuracy you want.

2. The latitude quadrant

Adjust the latitude scale so, that the 90^o reading is at right angles to the sight line. This is the equinox position. Imagine that you are in the North Pole at equinox. Then the Sun is in the horizon at noon, its altitude is 0^o, and your latitude is 90^o, of course. Make a midday mark at 60^o in the latitude scale and the equinox mark opposite it in the date scale. To find the position of the latitude scale for other dates, subtract Sun's declination for the date from 60^o. Without moving the scales from the equinox position, mark the date opposite the resulting angle in the date scale. For example, the reading for 20.11. is 60^o - (-20 ^{o}) = 80^o. Mark this date opposite the 80^o reading in the latitude scale. Do the same for enough dates to make the date scale reasonably accurate. The choice of 60^o for the midday mark is arbitrary, but it places the date scale in a sensible position.

References

- Karttunen – Donner – Kröger – Oja – Poutanen: Tähtitieteen perusteet, revised edition, Vaasa 1995 (the refraction curve on the latitude scale of the quadrant).
- Maailmanhistorian suuret löytöretket, Valitut Palat 1980 (the insert in Fig. 1).