

ABOUT PARALLAX AND... CONSTELLATIONS

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Abstract

In the first part of this workshop we shall discover how to measure the distance between the observer and a distant object with the parallax method. This will be done in a very practical way, using two images taken with a digital camera from two different close points. It will be important to show that the accuracy of the result depends on many factors and can be improved a lot ...

In 1989, a satellite called “Hipparcos” was launched to obtain parallaxes and proper motion of nearby stars, with a breathtaking accuracy of some *milliarcseconds*. The final Hipparcos Catalogue (120 000 stars) and the final Tycho Catalogue were completed in 1996 and used to create the Millennium Star Atlas, whose data are available through the Internet.

In the second part of this workshop, we shall collect these data about the stars of a well-known constellation (Ursa Major or Leo for example) and use them to determine the movement of each of its stars and deduce the shape of the asterism in the past or in the future...

Two examples of 3D evolutive model of a constellation will be then suggested.

INTRODUCTION

About parallax

Parallax, more accurately called *motion parallax*, is the change of position of an object against the background when it is seen by a moving observer: as seen in Figure 1, it is the apparent shift of an object that is caused by a small change in the observer’s position.

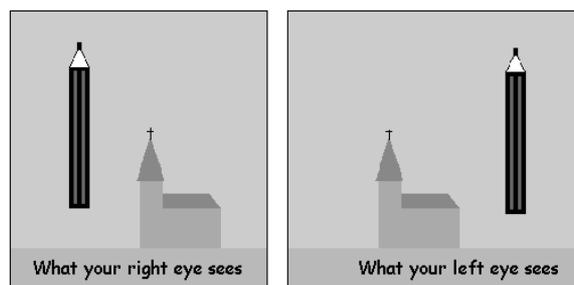


Figure 1

Hold a pencil in front of you and close your eyes alternately. When viewed with your right eye, the object appears to be at the left side of the church. If the viewpoint is changed to the left eye, the object *appears* to have moved to the right side of the church. By observing parallax, comparing angles and using geometry, you can determine the distance to various objects, especially astronomical ones.

Parallax can be used to determine the distance to the Moon: take two pictures of the Moon at exactly the same time from two locations on Earth, and compare the relative position of the Moon in relation to the visible stars. Using the orientation of the Earth, and the distance between these two points, a distance to the Moon can be computed.

In the case of stars, the effect is known as *stellar parallax*. The first successful measurements of stellar parallax were made by Friedrich BESSEL in 1838, for the star 61 Cygni.

Distance measurement by parallax is a special case of the principle of triangulation which states that one can calculate all the sides of a triangle if, in addition to all the angles, the length of one side has been measured. In parallax, the triangle is extremely long and narrow, and by measuring its shortest side AB (the motion of the observer) and the small top angle π (the other two being close to 90 degrees), the length D of the long sides (in practice considered to be equal) can be determined. (Figure 2)

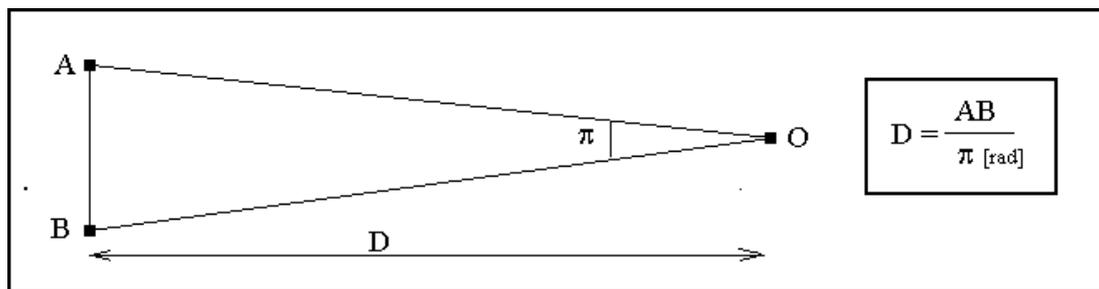


Figure 2

HIPPARCOS mission

Hipparcos (an acronym for **H**igh **P**recision **P**arallax **C**ollecting **S**atellite) was an astrometry mission of the European Space Agency (ESA) dedicated to the measurement of stellar parallax and the proper motions of stars. The project was named in honour of Hipparchus. Ideas behind such a mission dated from 1967, with the mission accepted by ESA in 1980. The satellite was launched by Ariane 4 on August 8th 1989.

The program was divided in two parts: the *Hipparcos experiment* whose goal was to measure the five astrometric parameters of some 120,000 stars to a precision of some 2 to 4 milliarcseconds and the *Tycho experiment*, whose goal was the measurement of the astrometric and two-colour photometric properties of some 400,000 additional stars to a somewhat lower precision.

ESA's Hipparcos mission exceeded all expectations and catalogued more than 100,000 stars to very high precision, and more than 1 million to lesser precision. Hipparcos was

so sensitive that it could have measured the diameter of a human hair at a distance of 20 kilometres. The mission produced 16 volumes of data.

The final Hipparcos Catalogue (120,000 stars with 1 milliarcsec level astrometry) and the final Tycho Catalogue (more than one million stars with 20-30 milliarcsec astrometry and two-colour photometry) were completed in August 1996. The catalogues were published by ESA in June 1997.

In this workshop, we will use these data, taking them from the Strasbourg Observatory website: <http://vizier.u-strasbg.fr/viz-bin/VizieR>

On the website:

- Choose the “HIPPARCOS” catalogue.
- Select “The Hipparcos Main Catalogue”.
- In “Query by Position on the Sky” you can indicate the name of the chosen star or its number in the Hipparcos catalogue: when you get them from the Internet, star lists generally provide this information.

GAIA project

Since then, the technology has improved. Detectors are better. On-board data handling, with the advent of more powerful computer processors, offers more possibilities. Optics have improved a lot. Scientists realised that another mission could be sent into space with cataloguing aims similar to *Hipparcos* but with more ambitious payback. A new mission could now catalogue one thousand million stars. It would have the equivalent sensitivity to measure the diameter of a human hair at 1,000 kilometres!

The successor mission to *Hipparcos*, *Gaia*, was approved in 2000 as an ESA Cornerstone mission to be launched around 2011.

Gaia will compile a catalogue of approximately one billion stars to magnitude 20. It will create an extremely precise three-dimensional map of stars throughout our Milky Way galaxy and beyond, and map their motions that encode the origin and subsequent evolution of the Milky Way.

SOME DETAILS ABOUT THE PARALLAX METHOD

General presentation

Aiming at object O from two points of view, A and B, we want to determine the distance from M (the middle of segment AB) to O (Figure 3).

We assume that the length of segment AB is known with a good accuracy and that its direction is perpendicular to line MO.

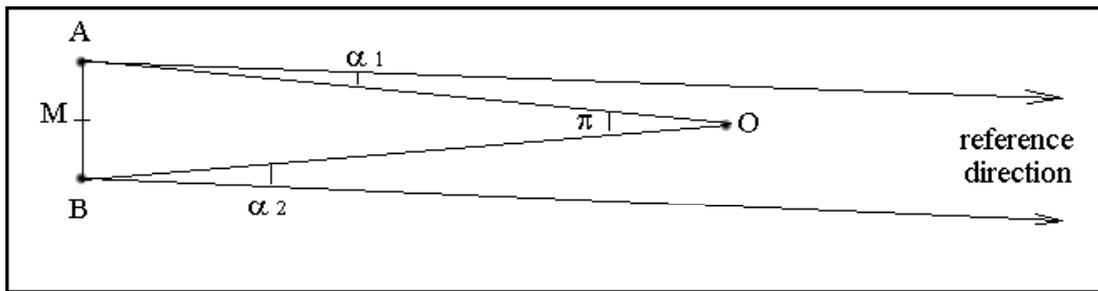


Figure 3

Measuring angles α_1 and α_2 , relative to a reference direction (given by a very distant object), it is therefore easy to deduce the value of the angle π , called *parallax* of point O as seen from A and B: In this case, $\pi = (\alpha_1 + \alpha_2)$.

As the length of segment AB is known, it is easy to calculate the distance OM. Generally, AB is very small comparatively to OM, so that $OA \cong OB \cong OM$.

In that case, π [rad] is a very small angle and we can write:

$$AB = OM \times \pi \quad \text{So: } OM = AB / \pi$$

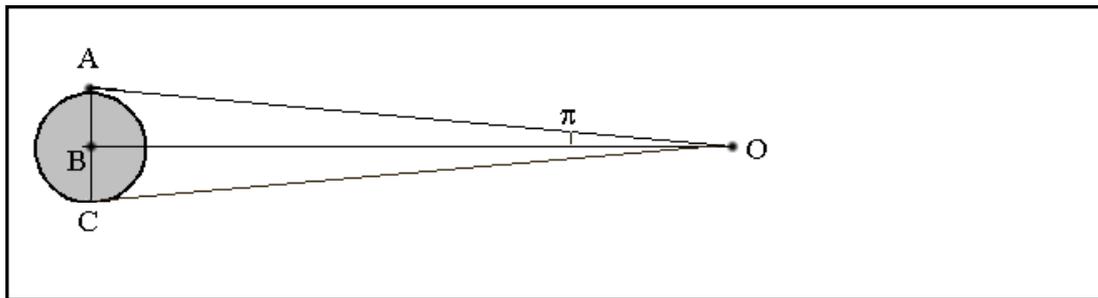


Figure 4

If points A and B are respectively a point on the Earth's surface and the Earth's centre, (Figure 4) π is called *diurnal parallax*, i.e., the angle subtended at the object (the Moon, a planet...) by the Earth's mean radius.

If points A and B are respectively the Earth's and the Sun's centres, π is called *annual parallax*, i.e., the angle subtended at the object (generally a star) by the mean radius of the Earth's orbit around the Sun.

Note that it is easier to do the measurements from points A and C and get the parallax as half the measured angle.

The *parsec* is defined as the distance for which the annual parallax is 1 arcsecond: one parsec equals 3.26 light years [ly].

The distance of an object (in parsecs) can be roughly computed as the reciprocal of the parallax. For instance, the Hipparcos satellite measured the parallax of the nearest star,

Proxima Centauri, as 0.77233 seconds of arc ($\pm 0.00242''$). Therefore, the distance is $1/0.772 = 1.29$ parsecs or about 4.22 light years (± 0.01 ly).

A PRACTICAL WAY TO USE THE PARALLAX METHOD

As a practical exercise, we shall measure the distance to a near object by the parallax method.

1. Choose this object so that it can be seen with a remote background.
2. Install your camera and note its exact position.
3. Take a first photograph of the object and some remote reference objects.
4. Move your camera perpendicularly to the aiming line and measure its displacement exactly. (Do not change the focal distance! do not modify the zoom!).
5. Take the second photograph.
6. Take a photograph of a graduated object situated at a well-known distance from the objective: it will be used as a reference for comparing angles.
7. Calculate the distance to the object!
8. Estimate the accuracy of your calculation.

The experiment

In order to calculate the distance to a small object (in this case a glue stick), placed in front of a remote mountain background, we have used two photos (Figure 5a and 5b) taken from two different points of view: in the case of our experiment, the distance between these two points was 70.0 cm.

In order to calibrate the images, an image of a graduated tape measure, situated at 1 metre from the camera was taken with the same camera. (Figure 5)

Optional information to verify the method: measuring the distance to the glue stick with a metre, we got 5.29 m...

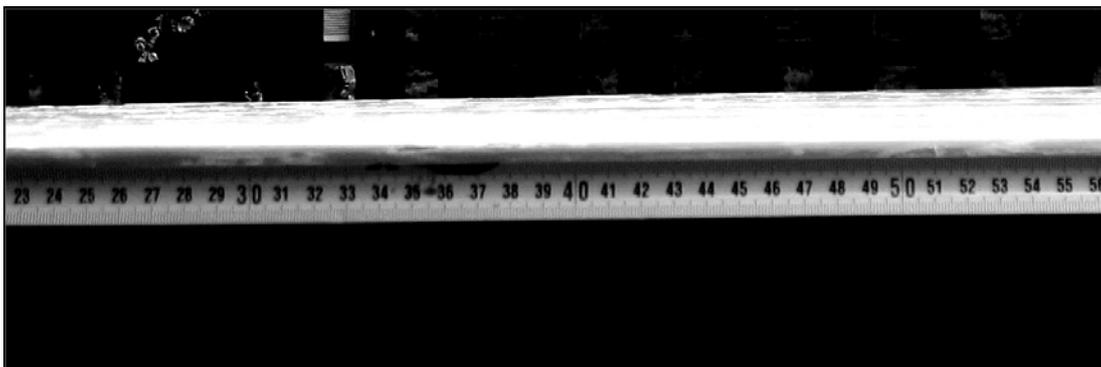


Figure 5



Figure 5a



Figure 5b

Calculations and results

One can calculate the scale of the printed photos, using the first image: 15 cm on the tape correspond to 6.5 cm on the photography. These 15 cm, as seen from 1 metre, appear under an angle = $\arctan (0.15 / 1) = 8.53^\circ$. Therefore, the scale of the printed photographs is: $8.53 / 6.5 = 1.31 \text{ }^\circ/\text{cm}$.

The apparent shift of the object relative to the mountain background can be measured using transparent paper: the mountain line can thus be drawn and the two positions of the glue stick can be drawn on the same paper. (Figure 6)

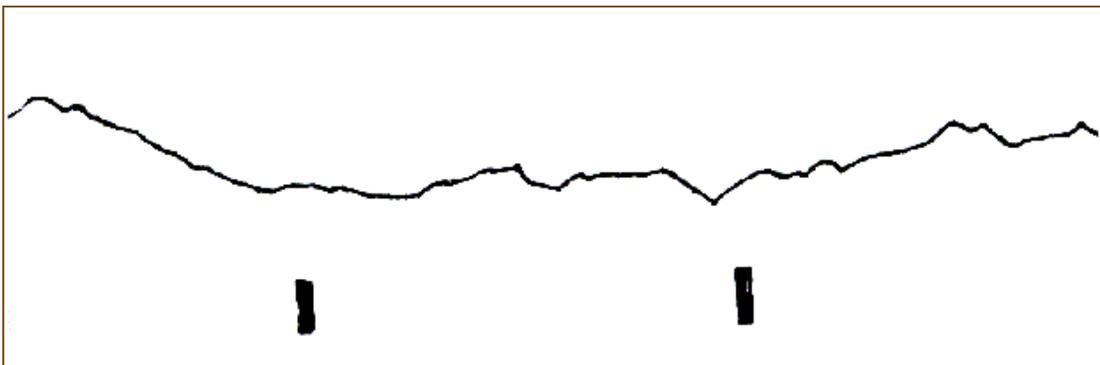


Figure 6

Then, it is easy to measure its apparent shift: We found 5.8 cm, that is to say a parallax angle of $5.8 \times 1.31 = 7.60^\circ$.

So, the distance X to the object is: $X = 0.700 / \tan (7.60) = 5.25 \text{ m}$... instead of 5.29 m as measured directly!

A method with easier calculations can be used: it consists in carrying directly the apparent shift on the calibration tape of the first photograph. The distance between the two sticks on the drawing corresponds to a “true” distance of 13.3 cm (0.133 m) on the tape measure, as seen from 1 metre.

So, a simple proportion can be written, using distance X as an unknown data:

$$\begin{array}{l} 1 \text{ m} \quad \longleftrightarrow \quad 0.133 \text{ m} \\ X \text{ m} \quad \longleftrightarrow \quad 0.700 \text{ m} \end{array}$$

$$X = 0.700 / 0.133 = 5.26 \text{ m}$$

As always, it is important to determine the accuracy of the measurement: The estimated distance between the two shifted images is between 13.2 cm and 13.4 cm. So:

$$X_{\min} = 0.700 / 0.134 = 5.22 \text{ m}$$

and:

$$X_{\max} = 0.700 / 0.132 = 5.30 \text{ m}$$

So we can deduce $X = 5.26 \pm 0.04 \text{ m}$, to be compared with the “true value” = 5.29 m.

ABOUT STARS AND PARALLAX

Here is a 20 years old text about a very well known constellation. Using the parallax data extracted from the *Hipparcos* catalogue, you will verify its present validity...

Ursa Major (Figure 7)

α UMa, called **Dubhe**, is situated at some **105 light years** (ly) from us.

β is called **Merak**, but the ancient Greeks called it **Helix**... It lays at **80 ly** and shines like 65 Suns.

γ is **Phekda**: at **90 ly**, it is coming towards us at only... 12 km/s!

δ is **Megrez**, the closest star of the Big Dipper: only **65 ly**...

ϵ , called **Alioth** is at **70 ly** and shines like 85 Suns.

ζ , that we call **Mizar** has a companion, **Alcor**. Both are situated at **88 ly** from the Earth.

η , at the end, is called **Alkaïd**: it is the most remote: **210 ly**...

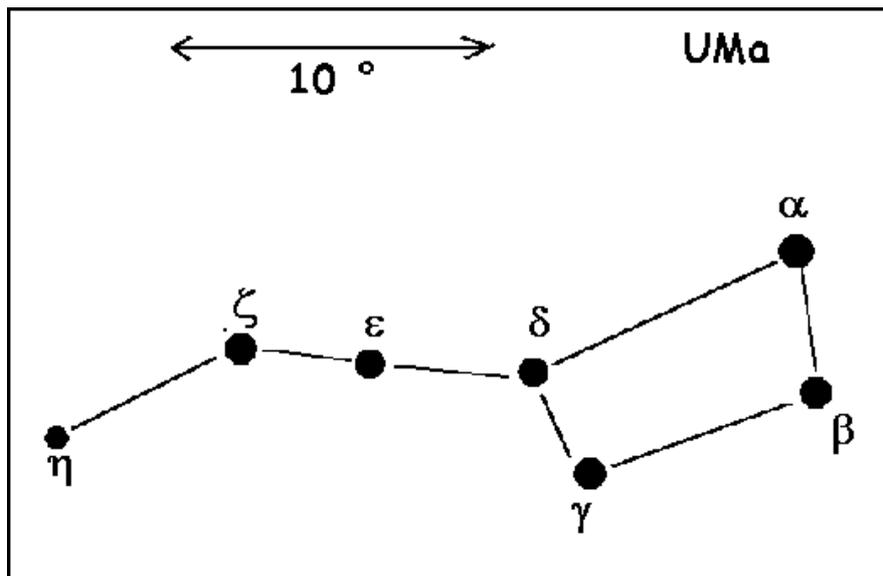


Figure 7

URSA MAJOR as seen by HIPPARCOS

Name	HIP	V Mag	RA (°)	DE (°)	pmRA (mas/year)	pmDE (mas/year)	Plx (mas)	V _{rad} (km/s)
Dubhe	54061	1.81	165.90	61.80	-136.46	-35.25	26.38 ± 0.53	- 8.9
Merak	53910	2.34	165.46	56.38	81.66	33.74	41.07 ± 0.60	-12.0
Phekda	58001	2.41	178.46	53.69	107.76	11.16	38.99 ± 0.68	-12.6
Megrez	59774	3.32	183.86	57.03	103.56	7.81	40.05 ± 0.60	-13.4
Alioth	62956	1.76	193.51	55.96	111.74	-8.99	40.30 ± 0.62	-9.3
Mizar	65378	2.23	200.98	54.92	121.23	-22.01	41.73 ± 0.61	?
Alkaïd	67301	1.85	206.89	49.31	-121.23	-15.56	32.39 ± 074	-10.9

Note: This table gives the *name* of the star, its *number* in the Hipparcos catalogue, its *visual magnitude*, *right ascension*, *declination*, *proper motions* (in *right ascension* and *declination*), its *annual parallax* and its *radial velocity*...

Exercise:

Using its *annual parallax* (Plx), deduce the distance D from each star to our Sun.

Remember!: The *parsec* is defined as the distance for which the annual parallax is 1 arcsecond. A parsec equals 3.26 light years [ly]... So: $D[\text{ly}] = 3.26 \times 1000 / \text{Plx} [\text{mas}]$.

ABOUT THE SHAPE OF CONSTELLATIONS

The Hipparcos Catalogue

Among the numerous information you can get from this star catalogue, we can consider the data about position and movement:

Using the equatorial coordinate system, *Right Ascension* (RA) and *Declination* (DE) are the coordinates of the star on the celestial sphere and allow us to place each star in the right place to construct the map of a constellation.

DE, expressed in degrees, is comparable to *latitude*, projected onto the celestial sphere, and is measured in degrees north and south of the celestial equator.

RA can be expressed in hours, minutes and seconds or, better for us, in degrees. It is the celestial equivalent of terrestrial longitude. Its zero point is known as the *Vernal point*, or γ point, the place in the sky where the Sun crosses the celestial equator at the spring equinox.

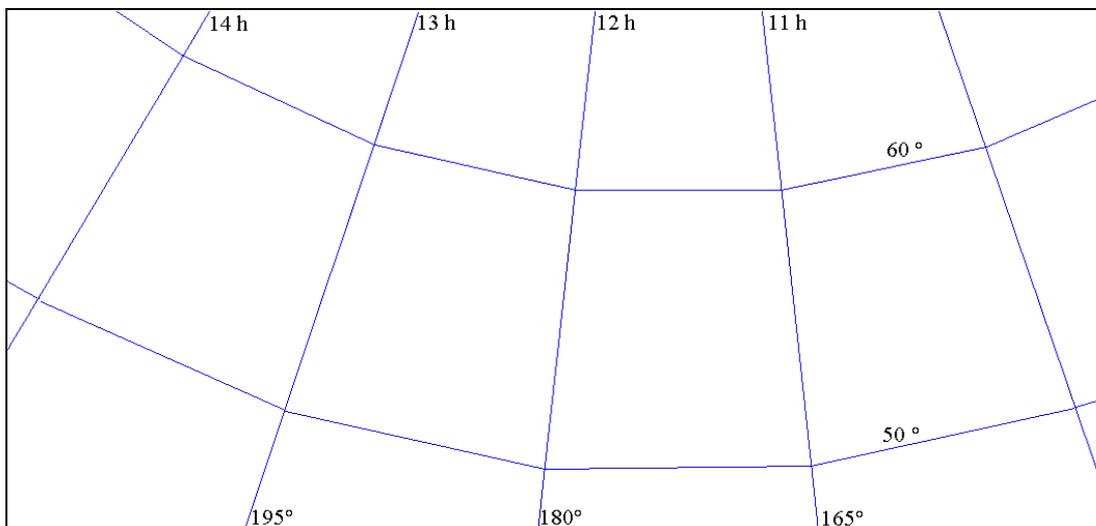
The stars are moving! At the human life scale, these movements are very faint and the stars seem still. But Hipparcos could detect the proper motion of the stars.

The coordinates of each star are changing slowly. Hipparcos gives two data for them: pmRA and pmDE indicates the *proper motion* of the star in Right Ascension and Declination. It is expressed in degrees by year. They allow us to determine the coordinates of a star at any moment in the past or the future, assuming that these angular speeds are constant.

The *radial velocity* (V_{rad}) of the star gives the speed [km/s] of the star along the Sun-Star line. A positive value indicates that the star is going away from us.

An investigation

- What will be the shape of the Big Dipper in 100,000 years?
- At first, place the 7 stars in their present position.
- Calculate for each of their coordinates in 100,000 years.
- Draw the future shape of the constellation.
- Find the distance of each star to the Sun [ly] nowadays and 100,000 years later.



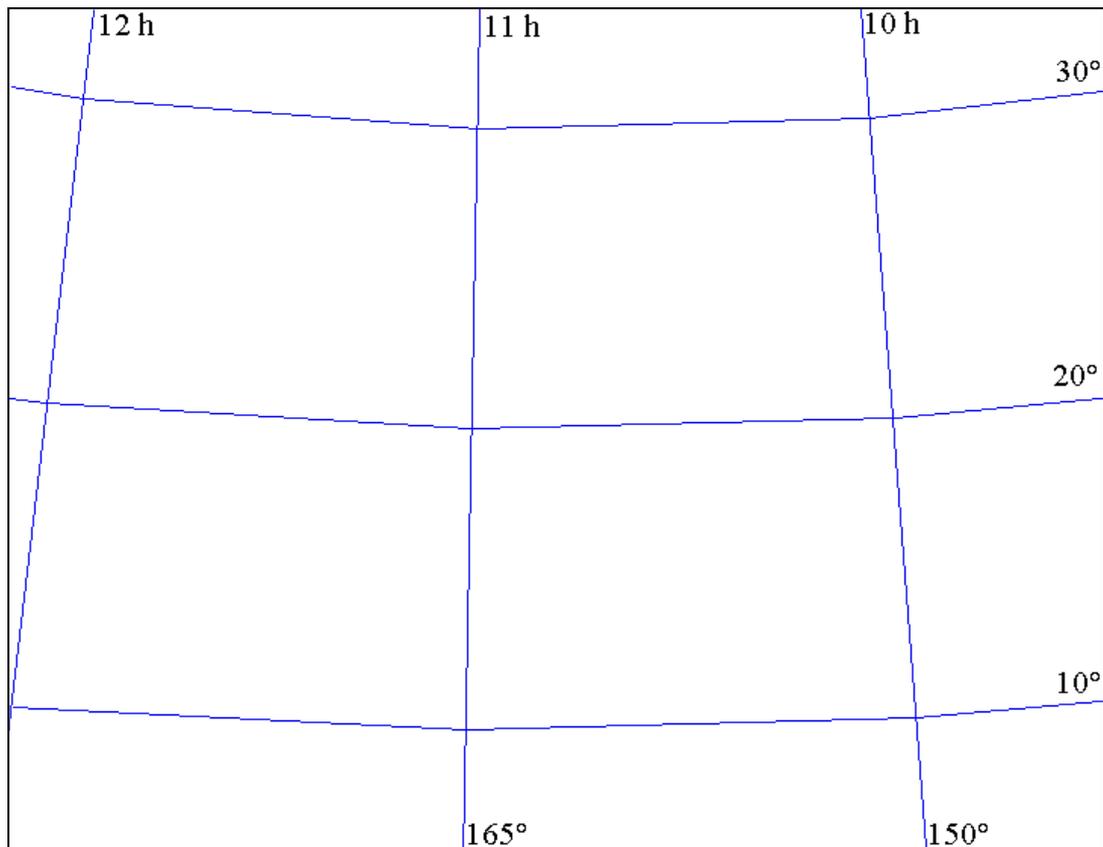
Remember!: The *light year* [ly] is the distance covered by light in one year.
 The speed of light is 300,000 km/s. If a star moves with a speed of 1 km/s, its displacement during a 100,000 year period will be: $1/3 = 0.33$ ly.

- And what will happen with Leo?

One can use the same method used for the Big Dipper

LEO

<i>Name</i>	<i>HIP</i>	<i>V Mag</i>	<i>RA (°)</i>	<i>DE (°)</i>	<i>pmRA</i> (<i>mas/year</i>)	<i>pmDE</i> (<i>mas/year</i>)	<i>Plx</i> (<i>mas</i>)	<i>V_{rad}</i> (<i>km/s</i>)
Regulus	49669	1.36	152.09	11.97	-249.40	4.91	42.09 ± 0.79	5.9
Denebola	57632	2.14	177.27	14.57	-499.02	-113.78	90.16 ± 0.89	36.2
Algieba	50583	2.01	154.99	19.84	310.77	-152.88	25.96 ± 0.83	?
Zosma	54872	2.56	168.53	20.52	143.31	-130.43	56.52 ± 0.83	-20.2
ε	47908	2.97	146.46	23.77	-46.09	-9.57	13.01 ± 0.88	4.3
ζ	50335	3.43	154.17	23.42	19.84	-7.30	12.56 ± 0.78	-15.6
η	49583	3.48	151.76	16.73	-1.94	-0.53	1.53 ± 0.77	3.3
θ	54879	3.33	168.56	15.43	-59.01	-79.37	18.36 ± 0.77	7.6
μ	48455	3.88	148.19	26.00	-216.29	-54.92	24.52 ± 0.87	13.8



A 3D-MODEL OF YOUR CONSTELLATION

With a drawing of the asterism that you want to represent, and strings coming from each star image to a same point, the Earth, we will put small balls representing each “real” star at the correct distance from the Earth... nowadays and in a remote future (100,000 years later)... You will get a 3D-evolutive-model of your constellation. Figure 8 is a provisional drawing showing the idea ... The real one will be better and more accurate!.

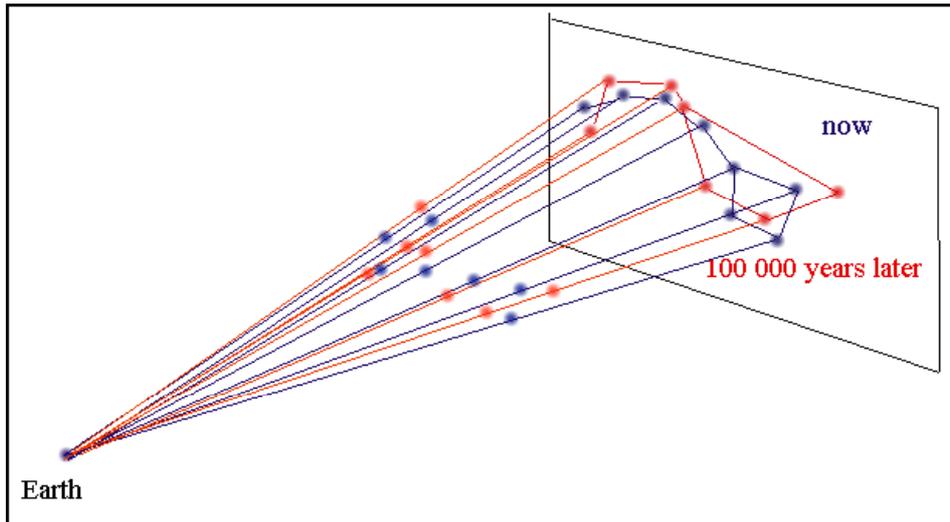


Figure 8

Cristina Palici di Suni suggests another possibility for a 3D model of parallax effect: it is the box shown in Figure 9. A teacher can prepare it in order to let the pupils understand that the parallax effect modifies the shape of a “constellation” of little balls on sticks if the point of view is moved from an up position to another one only a few centimetres down. As you can see in the picture where the box is open, there is a side of the box with a big window/hole to let the light come in when you close the box.

There are two holes to look in and 4 sticks with plasticine balls. One of them is fixed on the opposite side of the holes to look in, the others are on the floor of the box distributed in a way so that when you move the position of seeing two of them become higher or lower than the other two. The two different “constellations” become a sort of “corona” with the concavity reversed. It is necessary to calibrate the quantity of light coming in when you close the box because the effect have to be soft (you can glue a translucent paper on, if the light is too much).

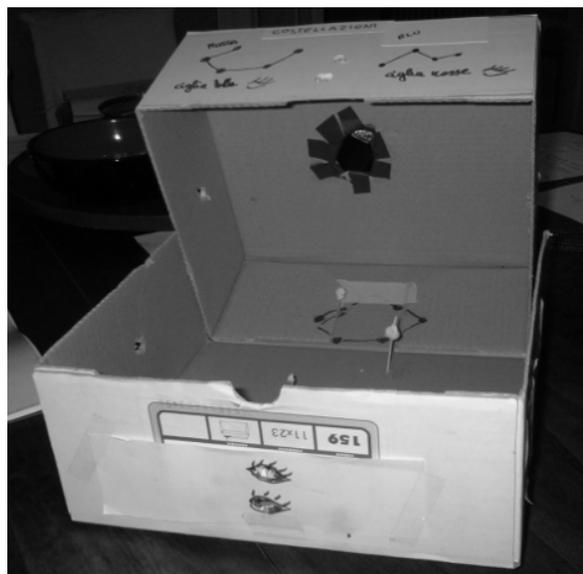


Figure 9

References

- HIPPARCOS catalogue: <http://vizier.u-strasbg.fr/viz-bin/VizieR>
- About HIPPARCOS mission: <http://www.rssd.esa.int/Hipparcos/catalog.htm>
- About GAIA Mission:
 - <http://www.esa.int/science/gaia>
 - http://en.wikipedia.org/wiki/Gaia_mission