WHERE ARE YOU? LOCATING BY THE SUN

Sakari Ekko

EAAE Summer School Working Group (Finland)

Abstract

In this workshop, we first find our location on the globe by the Sun and Polaris, the Pole Star. After that, we determine the coordinates of another location in relation to our position using a gnomon and timing with the help of a mobile phone. We use a big gym ball to demonstrate the idea, marking the positions and meridians on it. Students handle the conceptions of latitude, longitude, time difference, declination and altitude of the Sun.

INTRODUCTION

Columbus dared to sail west for Asia, because he had a wrong idea of Earth's size. He was very lucky: America was in the way, and he and his crew did not perish in the immense Pacific. Columbus did not know exactly where he was in East-West direction either. He was an excellent sailor and navigator, but he had no means to find his accurate longitude. It was a common – and for many sailors fatal - problem before the invention of the chronometer by Harrison in 18^{th} century. We will look at the problem more closely, and solve it using a piece of modern technology – mobile phone.

LATITUDE

Let us suppose we know that Earth is a sphere. We define a point on the surface of Earth using a grid of latitude parallels and longitude meridians expressed in degrees (Fig. 2). We can easily find, that the sun makes one revolution in the sky in 24 hours, as Columbus knew (of course the Earth is moving, but that Columbus did not know). Observing the sky, we would soon find that the Polaris is almost fixed in the sky throughout the night and year. Thus, the Celestial North Pole (CNP) is near the Polaris. Polaris will be higher and higher in the sky, when we go to the North, and vice versa, Figure 1. In fact, the altitude of Polaris, or more accurately, the altitude of the CNP, is the same as our latitude, ϕ in Figure 1.

We have only to measure the altitude of Polaris to get our approximate latitude. We can do it with a quadrant (Figs. 1 and 7), cross-staff or sextant. If we measure the altitude of Polaris in the evening and morning and use the average reading, the result is more accurate, because Polaris is about 0.8^o from the CNP. (In Columbus' time, it was about 4^o from CNP due to precession).

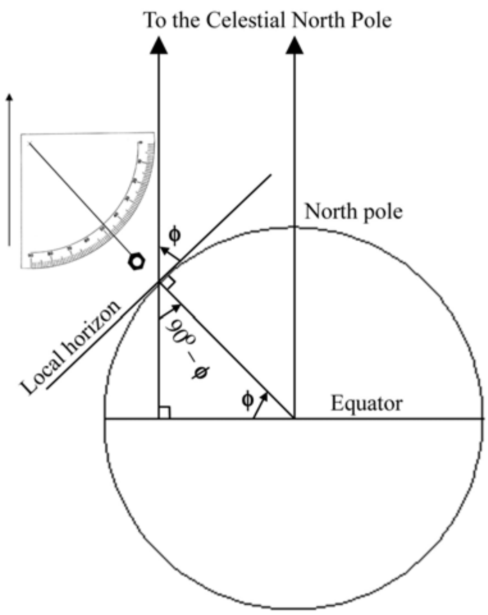

Figure 1. Measuring your latitude

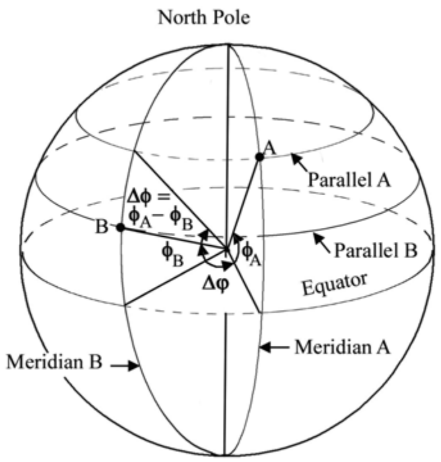

Figure 2. The longitude and latitude difference, $\Delta\varphi$ and $\Delta\phi = \phi_A - \phi_{B,}$ respectively

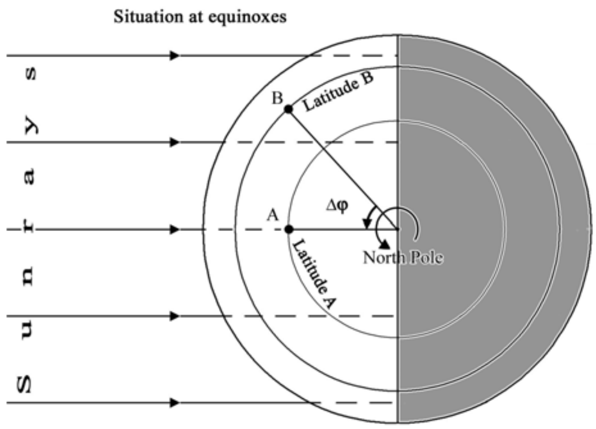

Figure 3. Earth seen from above the North Pole at equinox, with the Sun seen in south from location A. For determining $\Delta\varphi$ at other dates the geometry is essentially the same

LONGITUDE

We measure the latitude from the equator, but there is no natural zero meridian for longitude. We can measure the longitude difference between two places (A and B in Figs. 2 and 3), but there are no absolute longitude values. We can decide that one meridian, for example the one drawn through A, is the zero or prime meridian, and the longitude of another place is measured in degrees to the west or east of this meridian. By convention, we use the meridian going through Greenwich as a universal prime meridian.

Measuring the longitude difference is easy – in theory. Look at Figures 2 and 3. The Earth makes one full revolution in 24 hours and thus rotates $1/24 \times 360^o = 15^o$ in one hour. If we observe the moments, when the Sun is south in location A and B (T_A and T_B, respectively), the longitude difference of A and B is

$$\Delta\varphi = (T_B - T_A) \times 15^o, \text{ where } T_A \text{ and } T_B \text{ are expressed in hours.}$$

If the noon is later in B than in A, B is west of A, and vice versa.

So, why Columbus and countless navigators before and after him could not determine their longitude? Because they had no accurate clocks. Observing the Sun in A, they had no means to know, what the time of local noon in B was.

A sky phenomenon that can be seen from A and B can be used for timing. Columbus used a lunar eclipse in two occasions. His results were over 20^o west from his real position, helping to confirm his belief that he was in Asia.

DEMONSTRATING THE LATITUDE AND LONGITUDE DIFFERENCE

A 60 – 90 cm diameter gym ball is a usable model of the globe. Use a round biscuit tin without lid as a base, and turn the ball so, that its axis is inclined $90^o - \phi$ from vertical and points north (Figs. 4 and 5). Use tape as meridians and mark them at 10^o intervals (divide the distance from pole to equator by 9). Make two small gnomons, and tape them in locations A and B. Rest you find in Figures 4 and 5.

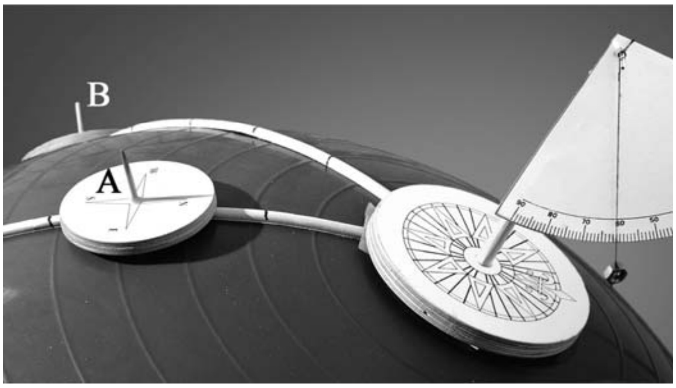

Figure 4. The gym ball Earth, with axis pointing to Polaris. Time T_A, noon at A, 61^o N. A is on the highest point of the ball. The shadow points to the north. The noon shadow of the gnomon in A is longer than in B, Fig. 5. Note: the degree scale shows the latitude

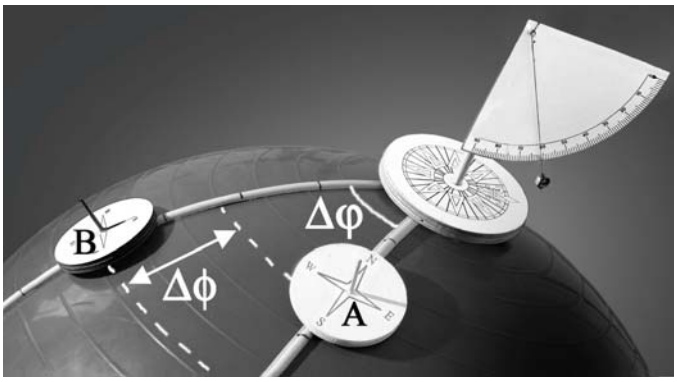

Figure 5. At your noon in A, turn the ball without moving its axis until the shadow of the gnomon in B points north. This demonstrates time T_B, noon at B, in reality happening some hours after the noon in A. B is $\Delta\varphi$ degrees west of A. $\Delta\varphi$ is the longitude difference, and $\Delta\phi$ the latitude difference

MEASUREMENTS IN PRACTICE

Find a cooperating class in another country. The idea is to find the latitude and longitude difference of the two schools, and with the help of Polaris, the absolute latitudes. As a prime meridian (zero meridian) you can use one of the two meridians going through the schools – as I mentioned above, the prime meridian is a question of convention.

First, use a quadrant to find the altitude of the pole star. The students can build a quadrant by themselves with help of Figure 7. Ideally, the measurement should be done in the evening and morning, 12 hours apart, and the mean value used, because Polaris is not exactly at the celestial North Pole. The result ϕ_A is the latitude of A.

Next, build a gnomon of a 20 cm (= h) long stick. Sharpen one end, round the other end of your gnomon and drill a suitable hole perpendicularly in a piece of wood or support the stick with modeling clay. Push the sharp end into the hole or modeling clay and place the gnomon on a sheet of cardboard or paper on level ground (Fig. 6). The gnomon must be vertical. Tap the stick lightly to punch a dent in the cardboard, and you have the base of your gnomon marked – that is why the stick was sharpened. Let us call this point C.

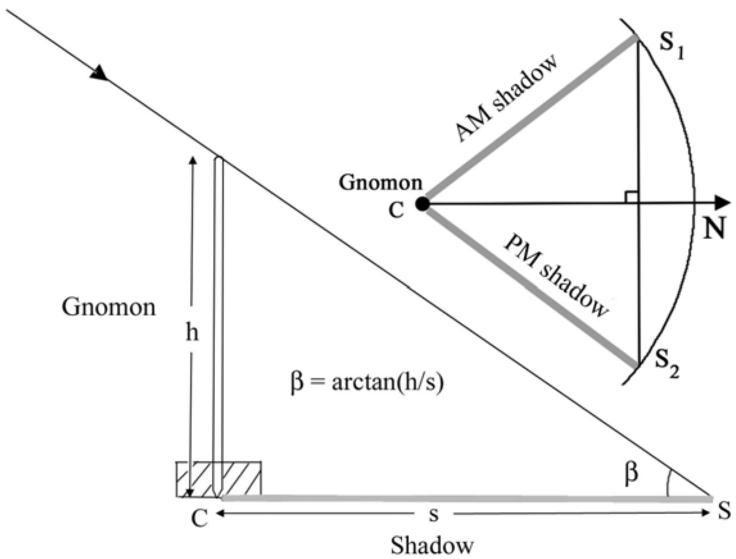

Figure 6. Calculating the altitude of the Sun. If your sudents are not familiar with trigonometry, they can draw the triangle and measure the altitude of the Sun from the triangle

The class in B makes the same preparations.

First, you have to find the North-South-direction. You can mark it in the night using Polaris, but you can use the Sun too. Look at Figure 6. Mark the tip of the gnomon shadow at, say, 10 o'clock (point S_1), take the gnomon out and draw a circle in C with radius CS_1. With the gnomon back in the right place, wait some hours until the shadow tip touches the circle again and mark it (point $S_{2)}$, noting the time. Without moving the cardboard, draw a perpendicular from B to the segment S_1S_2 , and you have the N-S direction. Leave the cardboard as it is, or use some distant object as a N-S landmark.

Next sunny day, place your gnomon on the cardboard with its N-S line aligned after your landmark. Measure the shadow length (s) when it points exactly north and note the time. This time is your local noontime. Call the cooperating class B at the moment of noon and give time to them: “Our noon is … NOW”. Then tell them your shadow length at noon.

The class at B make the same observations on the same day (let’s hope they have a sunny day too). They phone you at the time of their local noon, telling the moment and shadow length to you. Note the time your watch shows (T_B), and subtract your observed noontime (T_A) from it. Agree on several observing dates with class B in case of cloudy weather.

The longitude difference between A and B is $\Delta\varphi = (T_B - T_A) \times 15^o/h$, Figure 3.

If the difference is negative, B is west of A, if it is positive, B is east of A.
The altitude of Sun is:

$$\beta = \text{arc tan } (h/s)$$

The difference in latitude is $\Delta\phi = \beta_B - \beta_A$, and the latitude of B is $\phi_B = \phi_A - \Delta\phi$, Figure 2.

Take a globe or map and, using $\Delta\varphi$ and ϕ_B, try to find where B is.

If you find timing with mobile phone too prone to communication problems, you can simply e-mail the time and shadow length. Your friends do the same. Give the time in universal time (UT) or tell your time zone in the message.

Discuss the accuracy of the method with your students. For example, the tip of the gnomon shadow is not sharp. Why? How does this affect the latitude measurements?

EXAMPLE

In Turku, Finland (location A), we measure the altitude of Polaris. The result is 60.5^o. The Sun is south at 12.41 (= T_A) on 2nd of March. Our 20 cm gnomon casts a 48 cm long shadow at that moment (our local noon). We phone B just before the noon, giving our noon timing and shadow length.

Our friends in B phone us their noon timing and shadow length. Our clock shows 13.58 (=T_B) at the noon of B. The noon shadow length at B is 23 cm; the height of their gnomon is the same as ours, 20 cm.

Noon in B occurs 13h 58min – 12h 41min = 1h 17min = 1.28 h later than in Turku.

B is 1.28 h × 15°/h = 19.2° W of us; its longitude is 19.2° W, if we use our meridian as a prime meridian.

The altitude of the Sun at noon on 2nd of March is:

$$\beta_A = \arctan(20cm/48cm) = 22.6^o \quad \text{(in A)}$$
$$\beta_B = \arctan(20cm/23cm) = 41.0^o \quad \text{(in B)}$$

The latitude difference is $\Delta\phi = 41.0^o - 22.6^o = 18.4^o$

The latitude of B is 60.5° – 18.4° = 42.1°N.

The Greenwich-based longitude of Turku is 22.3° E, so B is 22.3° – 19.2° = 3.1° E from the Greenwich prime meridian.

B is at 42.1°N, 3.1° E. Look in a map or globe, where our friends are.

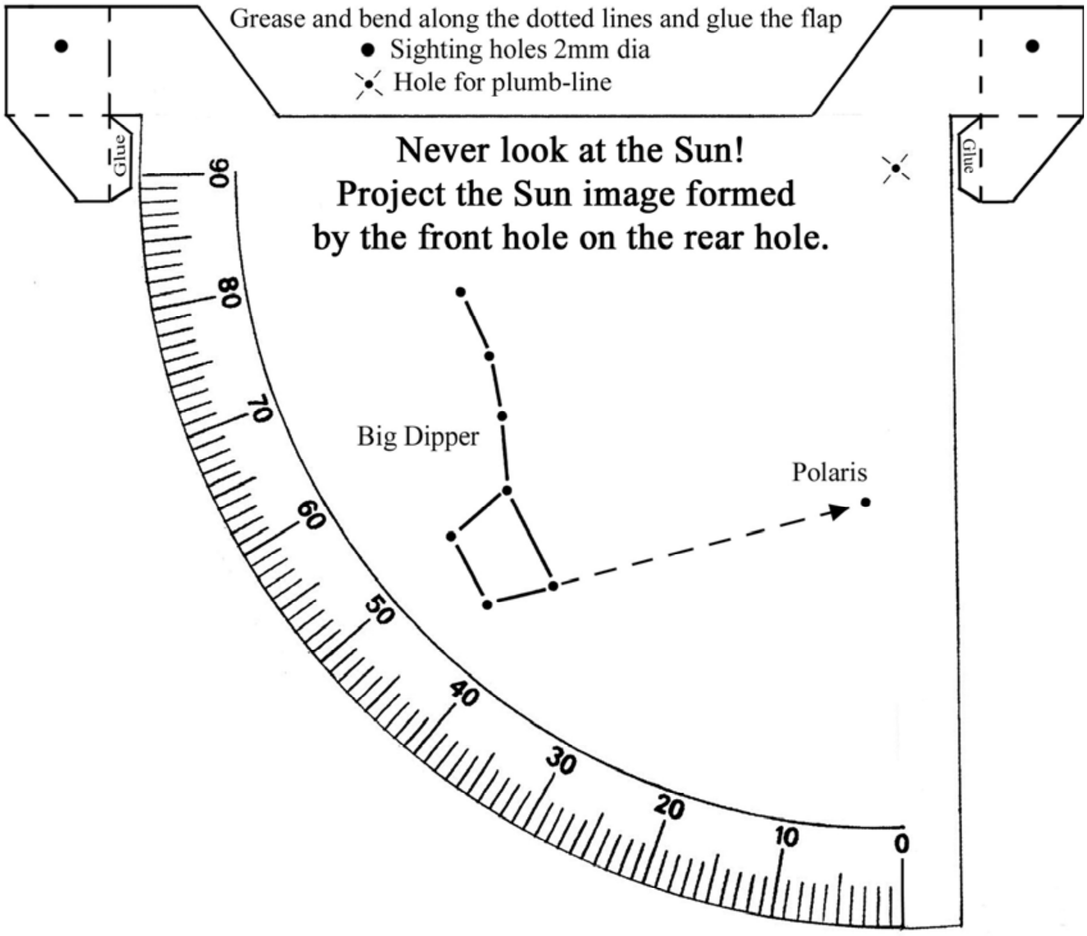

Figure 7. A simple quadrant. Use a thread and nut for plumb line. Warning: Never look at the sun! Instead, project the Sun's image formed by the front sight hole on the rear hole

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